The Tradeoff between Supported vs. Hanging Vehicles

J. E. Anderson

One of the most difficult tradeoffs in the design of PRT systems is the choice between use of supported or hanging vehicles, i.e., Supported-Vehicle Systems (SVS) or Hanging-Vehicle Systems (HVS). The DEMAG+MBB group[1] solved this problem by developing a guideway that permits one set of vehicles to ride above the guideway and another set that ride below. In my textbook[2]I examined the tradeoff between systems using one-way guideways and two-way, above-below guideways, and found that the cost per passenger-mile of the one-way system was somewhat lower. The two-way system reduces circuity[3] while riding the system, but to make the use of larger, two-way guideways economical the lines must be spread farther apart, which results in longer walking distances, which add more to the trip time than the two-way system reduces it. The two-way guideway had about twice the bulk of the one-way guideway, which increases visual impact and cost. I thus concluded that it is better to concentrate on one-way-guideway systems.

Number 21 of a series of 38 Frequently Asked Questions or FAQs that I wrote in the late 1980s considers this tradeoff. That series can be found on older versions of www.taxi2000.com by going to www.archive.org where one can retrieve a huge number of old web pages. These FAQs can also be found on www.acprt.org and www.prtnz.com. Under FAQ #21, I discussed ten considerations that led me to the conclusion that I should place the vehicle above the guideway rather than below. Almost all of them can be understood from the information given, but one consideration – torsion in curves – requires the reader to accept my word for a piece of analysis not given. The purpose of this paper is to provide a detailed derivation of the reason why torsion in curves increases the weight per unit length of SVS by a surprisingly small amount compared to HVS and why to obtain the same guideway natural frequency, the weight per unit length of an HVS guideway will increase by substantially more compared with SVS than it will be decreased by the consideration of torsion.

Requirement for super-elevation. To minimize curve radii the guideway of an SVS must often be super-elevated in curves, which increases the complexity and possibly the cost of manufacture. With an HVS, the vehicles can be permitted to swing as they round the curve up to an angle about twice that permissible with an SVS, thus supposedly eliminating the need for super-elevation. The swing of course must be damped to prevent the vehicle from rocking side by side, and that increases complexity. There is, however, a problem with too large a bank or super-elevation angle. It was uncovered in operation of the Swedish and British tilt trains: Their cars ride on tilting bogies, which permits them to bank in curves, thus permitting curves of fixed radii

[1] Development/Deployment Investigation of Cabintaxi/Cabinlift System, Report No. UMTA-MA-06-0067-77-02, NTIS Report No. PB277 184, 1977

[2] *Transit Systems Theory*, Lexington Books, 1978, Figure 5-8. (Available on www.advancedtransit.org)

[3] The ratio of distance along the guideway to the direct-line distance.

to be traversed at higher speed. It was found, however, that enough people got seasick in riding such trains that the maximum speed had to be reduced markedly over the desired speed. This problem will also be a factor in the design of PRT systems, which will limit the maximum bank angle and thus the advantage of using large bank angles to minimize curve radii, or with given curve radii to maximize speed.

Torsion in a Curved Guideway

Consider a vehicle composed of a cabin of weight W_{cabin} and a chassis of weight $W_{chassis}$ moving at speed V over a curved guideway with distance L between supports measured along the curved guideway. This geometry is depicted in Figure 1. The guideway is designed so that the radius of curvature at the center of the cabin is

$$R = \frac{V^2}{A_H}. \quad (1)$$

where A_H is the centripetal acceleration given by

$$A_H = g\tan\phi + \frac{A_l}{\cos\phi} \quad (2)$$

in which ϕ is the super-elevation or bank angle and A_l is the acceptable lateral acceleration felt by the passengers. With supported vehicles, we specify $\phi = 10^o$ and $A_l = 0.2g$, which gives $A_H = 0.379g$. With hanging vehicles it is usual to take ϕ large enough so that $A_l = 0$. If ϕ is increased enough so that A_H has the same value as with supported vehicles, then ϕ increases to 20.78^0. So we assume then, from equation (1), that R has the same value for both supported and hanging vehicles.

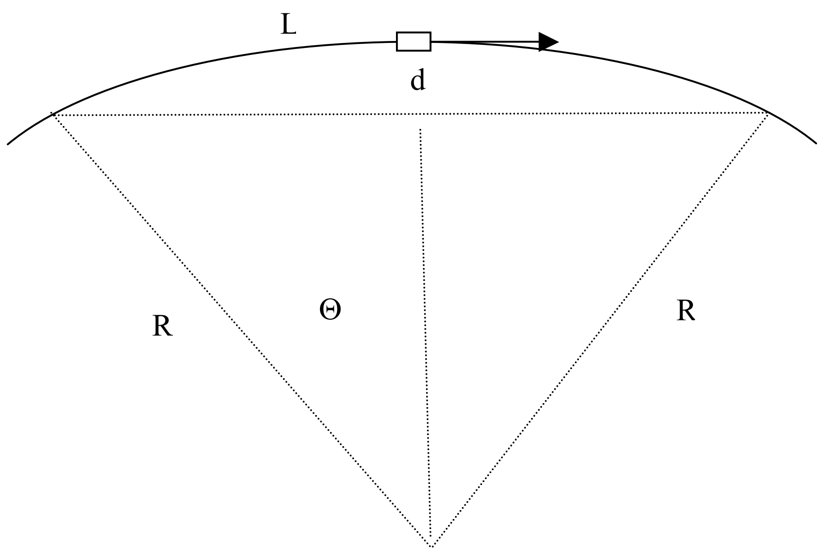

Figure 1. Vehicle moving on curved guideway half way between support posts.

Let Θ be the angle in Figure 1 measured from the center of curvature between a line directed to one of the support posts and a line half way between the posts. Then

$$\Theta = \frac{L}{2R_g}, \tag{3}$$

where R_g is the radius of the centerline of the guideway, which differs slightly from the radius calculated from equation (1), which is measured to the c. g. of the cabin. The distance between a point midway between two posts at the center of the guideway and a straight line connection a pair of posts is

$$d = R_g\left(1-\cos\Theta\right) = R_g\left[1-\cos\left(\frac{L}{2R_g}\right)\right] \tag{4}$$

From Figure 2, the mass center of the cabin of the vehicle is displaced from the center of twist of the guideway, but the mass center of the chassis is close to the center of twist of the guideway.

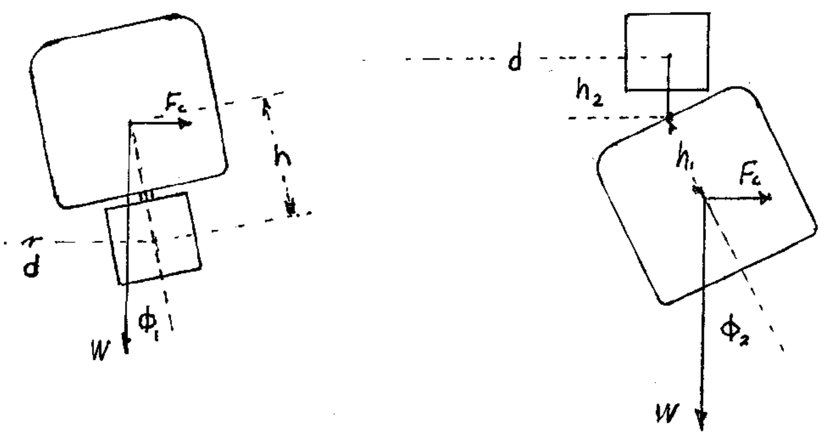

Figure 2. Centrifugal torque of supported or hanging vehicles

<u>Consider the supported vehicle.</u> In such a system the guideway will be superelevated at an angle shown in Figure 2 as ϕ_1. If the torsional moment at each post due to the moving vehicle is designated by T_s then from Figures 1 and 2

$$T_s = \frac{1}{2}\cos\Theta\left\{W_{chassis}d_s + W_{cabin}\left[d_s - h\sin\phi_1 + \frac{\left(g\tan\phi_1 + A_l/\cos\phi_1\right)}{g}h\cos\phi_1\right]\right\}$$
$$= \frac{1}{2}\cos\Theta\left(Wd_s + W_{cabin}h\frac{A_l}{g}\right) \qquad (5)$$

in which $d_s = d$ in equation (4) with

$$R_g = \frac{V^2}{A_H} + h\sin\phi_1 \qquad (6)$$

Also

$$W = W_{chassis} + W_{cabin} \qquad (7)$$

Consider the hanging vehicle. If the torsional moment at each post is designated by T_h then from Figure 2

$$T_h = \frac{1}{2}\cos\Theta\left\{W_{chassis}d_h + W_{cabin}\left[d_h + h_1\sin\phi_2 - \frac{g\tan\phi_2}{g}\left(h_2 + h_1\cos\phi_2\right)\right]\right\}$$
$$= \frac{1}{2}\cos\Theta\left(Wd_h - W_{cabin}h_2\tan\phi_2\right) \qquad (8)$$

in which in equation (4)

$$R_g = \frac{V^2}{A_H} - h_1\sin\phi_2 \qquad (9)$$

Thus, the ratio of torsional moments is

$$\frac{T_s}{T_h} = \frac{d_s + \frac{W_{cabin}}{W}h\frac{A_l}{g}}{d_h - \frac{W_{cabin}}{W}h_2\tan\phi_2} \qquad (10)$$

The Guideway-Weight Ratio

The twist angle of a beam of open section is given by the equation[4]

$$\theta = \frac{M_t}{\frac{1}{3}bc^3G} \qquad (11)$$

in which, in Timoshenko's notation,

M_t is the twisting moment

[4] S. Timoshenko, *Strength of Materials*, Part II, D. van Nostrand Company, Inc. 1941, equation (253), page 270.

b is the perimeter of the beam
c is the wall thickness
G is the shear modulus

So that the torsional stiffness of the supported and hanging guideways are the same, we must have the ratio M_t / θ the same for the two guideways. The shear modulus will of course be the same and it is reasonable to take b the same in the two cases. Then solving equation (11) for c we get

$$c = \left(\frac{3M_t}{\theta b G} \right)^{1/3} \quad \text{so} \quad \frac{c_s}{c_h} = \left(\frac{M_{t_s}}{M_{t_h}} \right)^{1/3} = \frac{W_s}{W_h} \tag{12}$$

i.e., the ratio of the weight per unit length for SVS to HVS is proportional to the cube root of the ratio of twisting moments. This weight ratio was calculated in an Excel spread sheet with the following results:

g =	9.80665 m/s^2			
Speed =	30 mph		or	13.41 m/s
Post Spacing =	90 ft		or	27.43 m
Phi, supported =	10 deg		or	0.1745 radians
Al =	0.2 g's		or	1.961 m/s^2
AH =	0.3794 g's		or	3.721 m/s^2
R =	158.59 ft		or	48.34 m
Phi, hanging =	20.78 deg		or	0.3626 radians

	Supported	Hanging		
h =	1	n/a	m	
h1 =	n/a	1	m	
h2 =	n/a	0.6	m	
Rg =	48.51	47.98	m	
d =	1.926	1.947	m	
Wcabin/W =	0.5	0.7		
Torsion Ratio =				1.13

Beam Weight Ratio, Supported/Hanging

Speed, mph ->	20	30	40
Post Spacing, ft			
60	1.04	1.11	1.21
90	1.01	1.04	1.08

Natural Frequency of the Guideway

If the vehicles are supported above the guideway, the guideway can be clamped to the posts, whereas if the vehicles hang from the guideway, the support posts must be placed at one side with a horizontal member extending from each post to the top of and to support the guideway. Because of the limited torsional rigidity that can practically be built into such support structures, the joints at the posts behave much more nearly like simple supports than clamped supports.

From Marks' Standard Handbook for Mechanical Engineers, 10th Edition, page 3-73, the natural frequency of a beam clamped at both ends is $1.506^2 = 2.268$ times the natural frequency of a simply supported beam of the same properties. Moreover, the natural frequency of any beam increases as the square root of the moment of inertia. Compared with a clamped beam then, to achieve the same natural frequency, a simply-supported beam must have a moment of inertia $2.268^2 = 5.144$ times the moment of inertia of a beam clamped at both ends.

As a simple example, consider a box-beam guideway in which the depth and width is the same for both the SVS and HVS. Then from an undergraduate textbook on Strength of Materials, one can readily determine that the moment of inertia of the guideway in bending is directly proportional to the wall thickness of the box beam. Hence the weight of the guideway per unit of length is proportional to the moment of inertia. We see then that taking into account both torsional rigidity and natural frequency, obtaining a sufficiently high natural frequency in bending has a much stronger influence on weight per unit length and hence cost per unit length than the requirement for adequate torsional rigidity. The bottom line is that torsion is not a reason to prefer HVS over SVS.