Journal of Advanced Transportation, Vol. 22, No. 2, 1988, pp 108-133.

What Determines Transit Energy Use?

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A comparison of the energy use per passenger-mile of eight modes of urban transportation is made in terms of eleven variables, resulting in conclusions about the direction transit system design should take to provide adequate transportation with minimum energy use. The method can easily be programmed on a personal computer to be used to study the effects of parameter variations on energy use.

Introduction

About one fourth of the purchased energy used in the United States goes directly into transportation, about 40 percent of which is expended in urban areas. There have been a substantial number of excellent papers such as Boyle (1983), Kulash (1982), McCoy (1982) and EPRI (1986) that discuss and tabulate the energy use of various modes of urban transportation. However, little information exists on the fundamental factors that determine energy use in a way that can help guide thinking about minimizing transportation energy consumption.

While today the world is awash in oil, serious analysts predict that the days of energy shortages will return. It is therefore important to think anew about a possible age of energy shortages and what it implies for transit system design. Even if no energy shortages occur, it makes no more sense to waste energy than to throw dollar bills out the window. Design optimization to provide better service at less cost and lower energy use is just as

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important in the transit field as elsewhere, and is much needed as the vast majority of public transportation still depends on concepts put into practice a century ago.

Urban transportation modes vary considerably in their overall energy intensity per passenger-mile. Because the energy requirement depends on a large number of parameters, understanding transit energy use is difficult. Tabulation of energy use is not of much help unless it is accompanied by an analysis of the parameters involved and how their variation affects overall energy use. This paper starts with the equation of transit energy use in a form that takes into account the major variations. The equation is applied to eight modes of urban transportation, making use, wherever possible, of average characteristics of real systems and it is shown how eleven basic parameters affect the results. Based on work of Levinson et. al. (1984), an estimate of the energy of construction for each mode is added. Having done this, it is then possible to discuss the implications for design of new systems. The method developed will ease the problem of estimating the energy use of any new transit system.

The Energy Equation

The energy equation used is derived in Appendix A, in which all of the parameters involved are listed at the beginning. In the derivation, I take into account acceleration and deceleration, but not rate of change of acceleration (jerk) because of its small effect. I assume that the vehicle cruises at line speed until the brakes are applied, i.e., I don't take into account a coast phase. The coast phase is often used in rapid rail systems; but, in these systems, it has only a small effect on overall energy use because the vast majority of direct energy goes into kinetic energy rather than into overcoming air drag or road resistance.

While regenerative braking is used in some transit systems, I don't include either regenerative braking or any energy used in braking. Regenerative braking isn't as useful as one would hope 1) because the kinetic energy attained at line speed is only a fraction of the energy input required to achieve it, and 2) because only a fraction of the actual kinetic energy can be recovered in braking. Knowing these fractions or efficiencies, however, enables one quite directly to study the effect of regenerative braking on overall energy efficiency.

Use of the energy equation A-5 requires determination of air drag and road-resistance coefficients. The air-drag coefficients were deduced for the various modes considered from Hoemer (1958), still the "bible" on aerodynamic drag. For the conventional modes, road-resistance formulae from Hay (1977) were used. For the new mode included, see Anderson (1988), here called "personal rail" in parallel with heavy rail and light rail, the road-resistance coefficients were derived from basic formulae contained in Clark (1981).

Energy equation A-5 contains both the line speed and the average speed. From the viewpoint of comparing systems, the average speed is the important variable because dividing trip distance by average speed gives trip time, perhaps the most fundamental service parameter. A formula for line speed is therefore derived in Appendix B in terms of average speed, trip distance, comfort level of acceleration, and station dwell time.

Note from equation A-5 that energy use per passenger-mile depends on several lumped parameters: gross weight per daily-average passenger, the effective frontal area $C_D A_f$ per passenger, auxiliary power for heating and cooling per passenger, and the kinetic energy per unit of weight and distance $V_L^2 / 2gD_s$. Energy also depends on line speed, wind speed, average speed, acceleration (through the parameter *k*), propulsion efficiency and utility or power-plant efficiency.

Modes of Urban Transportation

The eight modes of urban transportation compared in this paper are listed in Figure 1 along with the abbreviations used in the rest of the paper. The characteristics of these modes used in this paper are listed in Table 1. That is, for the purposes of the present analysis, each of the modes is defined by the values listed in Table 1. These characteristics are averages weighted by use, and in each mode certain of the characteristics vary over a wide range. This must be taken into account in interpreting the results. If the reader wants to investigate variations in parameters in some particular cases, he can do it by programming equations A-5 and B-4 and making as many runs as desired. That is the beauty of the computer age.

The first seven modes listed in Table 1 are conventional and have been in existence for 80 to 100 years with, notwithstanding advances in component technology, virtually no modification in their basic physical and service characteristics. Presently operational large-vehicle automated people movers that stop at stations on line are not included because, in energy use, they are very similar to electric streetcars and trolley buses. Also, data on them in the form needed is not so readily available. An eighth mode is included that, in this paper, is called "personal rail" in order to provide a two-word descriptor similar to heavy and light rail. This new mode, Anderson (1988), is an optimized version of personal rapid transit that the author began developing in 1981 after investigating the field since 1968.

MODES OF URBAN TRANSPORTATION

HR	**Heavy Rail Transit**
LR	**Light Rail Transit or Streetcar**
TB	**Trolley Bus**
MB	**Motor Bus**
VP	**Van Pool**
DB	**Dial-a-Bus**
A	**Automobile**
PR	**Personal Rail or Rapid Transit**

Figure 1. Modes of Urban Transportation.

Data Sources

The basic source of conventional transit data used was UMTA (1986), notwithstanding reservations concerning comments that there are certain reasons related to the rules for obtaining UMTA funds that sometimes cause transit operators to modify the data they report. In some cases, it is said, where they don't have the funds to obtain the required data, they guess. In spite of these doubts, the UMTA data is the only source found for the data needed, and glaring anomalies would be evident when substituted into the energy equation.

The UMTA data allow one to deduce the following ridership-weighted average parameters for the U. S. transit fleet: the vehicle capacity, the average speed for all but the van-pool mode, the daily average passengers per vehicle and hence the load factor (ratio of passengers to capacity), the yearly average kilowatt-hours per passenger-mile for electric systems, and the gallons of fuel per passenger-mile for the diesel or gasoline-driven vehicles. For heavy rail, light rail and trolley bus, the values for kWhr/pass-mi deduced from the UMTA data are listed at the bottom of Table 1 as "billed electrical energy." The values in the row directly above are obtained by dividing "billed electrical energy" by a power-plant efficiency of 31.9%. For the motor bus, van pool and dial-a-bus, the values in the second to last row of Table 1 "measured direct energy input" were obtained by converting the UMTA gallons of diesel fuel at 136,000 Btu/gal and 3412 Btu per kWhr. For the automobile, 20 miles per gallon of gasoline is assumed converting at 128,000 Btu/gal, as a representative value.

It is interesting to note that the data listed by UMTA on kWhr/pass-mi for each of the eleven U. S. heavy rail systems varies from highs of 1.39 for Miami and 1.37 for Baltimore to a low of 0.212 for the Philadelphia system, but that the New York system is so large that the U. S. average, weighted by passengers per year, is almost exactly the New York value of 0.305.

Table 1. Properties of Transit Systems

Property	Heavy Rail	Light Rail	Trolley Bus	Motor Bus	Van Pool	Dial-a-Bus	Auto	Personal Rail
Vehicle design capacity	189	117	74	58	16	13	6	3
Vehicle empty weight, lb	69930	44230	22870	16828	7760	6310	2500	860
Vehicle cabin length, ft	73.0	52.3	40.0	40.0	17.5	15.0	7.5	8.0
Vehicle width, ft	10.1	7.9	8.3	8.2	7.4	7.4	5.7	5.3
Vehicle height, ft	11.7	10.6	10.2	10.1	9.0	9.0	4.3	5.0
Frontal area, sq ft	118.2	83.7	84.7	82.8	66.6	66.6	24.5	26.5
Surface area, sq ft	3419	2103	1649	1630	707	625	199	218
Drag coefficient	0.12	0.51	0.52	0.46	0.58	0.60	0.35	0.40
Road resistance, lb =								
Wgt(a + bV), where								
a	0.0003	0.0003	0.007	0.007	0.009	0.009	0.009	0.004
b, 1/mph	0.000005	0.000005	0.000051	0.000068	0.000147	0.000215	0.000518	0.000439
Comfort Acceleration, g	0.125	0.125	0.125	0.125	0.25	0.25	0.25	0.25
Ave. Station Dwell, sec	40	45	30	15	10	60	41	8
Ave. Dist. Between stops, mi	0.489	0.252	0.244	0.249	1.990	0.400	0.342	2.400
Average speed, mph	17.9	11.0	8.2	12.8	43.8	14.0	12.6	22.5
Line speed, mph	40.6	39.2	12.2	18.7	49.6	40.6	23.6	23.2
Daily ave. pass./veh.	23.2	15.2	3.6	5.9	13.0	1.4	1.2	1.0
Daily average load factor	0.12	0.13	0.05	0.10	0.81	0.11	0.20	0.33
Empty weight/capacity, lb	370	378	309	290	485	485	417	287
Gross weight/passenger, lb	3149	3045	6488	2987	732	4642	2218	995
Effective area/pass, sq ft	0.61	2.81	12.23	6.46	2.97	28.54	7.15	10.60
Auxiliary energy/pass., kW	0.442	0.415	1.374	0.829	0.163	1.340	0.498	0.653
Kinetic energy, ft-lb/lb/ft	0.021	0.039	0.004	0.009	0.008	0.026	0.010	0.001
Energy terms in kWhr/pass-mi.								
Kinetic energy	0.835	1.460	0.310	0.330	0.071	1.507	0.284	0.018
Road resistance	0.019	0.014	0.617	0.304	0.172	1.101	0.705	0.233
Air drag	0.025	0.078	0.091	0.081	0.226	1.317	0.138	0.210
Heating & air conditioning	0.077	0.118	0.524	0.202	0.012	0.300	0.124	0.091
Total direct energy input	0.956	1.671	1.542	0.917	0.481	4.225	1.251	0.551
Construction energy	0.434	1.327	0.186	0.118	0.051	0.474	0.443	0.097
Total energy use	1.390	2.998	1.728	1.035	0.532	4.699	1.694	0.648
Measured direct energy input	0.956	1.671	1.542	0.917	0.481	4.225	1.251	
Billed electrical energy	0.305	0.553	0.492	—	—	—	—	0.176

Note #1: Conventional transit-system data are U. S. averages from the UMTA 1984 Section 15 Data Report.
Note #2: Electric-power-plant efficiency 31.9%, overall heat-to-work efficiency 16%.
Note #3: Drag coefficient for heavy rail takes into account four-vehicle consist.
Note #4: Energy content: 136,000 Btu/gal for diesel fuel; 128,000 Btu/gal for gasoline: 3.412 Btu = 1 W-hr
Note #5: Auto energy based on 20 mpg.
Note #6: Road resistance for Personal Rail from Clark (1980), othes from Hay (1983).
Note #7: Average passenger weight is 135 lb, HVAC requires 3 watts/sqft, 10 mph wind included.

Data on the length, width, height and weight of the seven conventional modes was obtained from Lea (1975) as averages interpolated to correspond to the average vehicle capacity obtained from the UMTA data. Data on these parameters for PR are for the Taxi 2000 system (Anderson 1988).

The comfort level of acceleration is taken from international standards as one-eighth g for vehicles permitting standing passengers, and one-quarter g in vehicles in which all adult passengers are seated.

An estimate was obtained for the heating and cooling power, a piece of information not found in the available data. Mean temperatures in the U. S. were used to estimate a seasonal average heating or cooling temperature difference of 25^{o} F. Based on a reasonable estimate of vehicle wall thicknesses and coefficients of thermal conductivity and considering window and door losses, a requirement of three watts per square foot for each of the eight modes was assumed. The parameter P_{aux} of Appendix A for each mode is then 3 W/sq-ft multiplied by the vehicle surface area. It is listed in Table 1 as auxiliary energy per passenger.

The construction-energy values listed in Table 1 for the seven conventional modes are values given by Levinson et.al. (1984), converted to the units used here. For HR, the value listed is for subways. For LR, it is taken as an average of the values listed by Levinson for grade-separated and surface-street construction because these systems usually require some of each. Since Levinson shows energy for grade-separated construction to be over seven times that for surface-street construction, there is considerable variation-in any actual situation. Construction energy for PR was developed from values of energy per ton of construction material and energy required to build vehicles of a given weight provided by Levinson (1984) and using data on quantities for the Taxi 2000 system.

Finally, as mentioned, a power-plant efficiency of 31.9% is assumed — a value often used in energy analyses. Efficiencies can go up to about 42% in modern plants, but the range of 30% to 33% is more common. Propulsion-efficiency differences between modes are not easy to determine. I therefore used an overall efficiency, propulsion efficiency multiplied by power-plant efficiency, for all modes of 16%. By inference, this means that for the electric systems, I assumed a propulsion efficiency of 0.16/0.319 or 50.2%. In this paper, these values are intended to be representative. Their reasonableness can be deduced from the results presented below.

Discussion of Results

Two parameters have not yet been discussed: the average distance between stops and the average station dwell time. The data source available provided no such information. What was done to make the comparisons, therefore, was to vary these values in the computer program until the total direct energy input for each mode (5th from the last row of data in Table 1) was equal to the 2nd from the last row. Thus the equality of these two rows of data is deliberate. This procedure is justified for this analysis because it gives a complete set of parameters that agrees with the reported measured energy use and gives a way of assessing the reasonableness of the results.

Note that, in spite of reports of faulty data, the distance between stops is in the right range. Average station dwell times are often as low as 15 seconds but in busy stations they are often much longer. Observed station dwell times often are as much as two or three minutes. For a given distance between stops and a given average speed, increasing station dwell increases the line speed, which, from Equation A-5 increases all of the energy terms. Thus, perhaps a smaller station dwell time implies a somewhat higher efficiency than assumed. In the case of van pools, average speed could not be deduced from the data. Therefore, given some knowledge of the kind of suburb-to-work service usually provided by commuter vans, three values — dwell time, distance between stops, and average speed — were chosen in reasonable ranges that would give the total measured direct energy input.

Figure 2 and through 10 illustrate graphically how specific parameters that enter equation A-5 vary. The numbers are all in Table 1. Figure 2 shows the wide range of vehicle capacities considered, but Figure 3 shows that the empty vehicle weight per unit of capacity varies surprisingly little. The term that enters the energy equation is gross vehicle weight per daily average passenger. To get it, divide the term of Figure 2 by load factor, shown in Figure 4, and then add the average passenger weight, which was taken to be 135 lb.

Load factor, therefore, becomes of direct importance in the kinetic-energy term and the road-resistance term. The low daily-average load factors of all of the general-service public systems (HR, LR, TB, MB) is inherent in the nature of the large-vehicle service concept. Economic studies such as Anderson (1984) show that the major reason for use of large vehicles is to amortize the wages of drivers over as many trips as possible. (Another important reason is to obtain reasonable throughputs at the required long headways.) The vehicle size must be chose to meet the rush-hour requirement. In slack periods, vehicle occupancy falls off so much that for reasonable economics fewer vehicles must be used. But if fewer vehicles are used, the schedule headway must increase, causing the ridership to drop off even more. Automation therefore has only a small advantage if, as is common today, the vehicle size is kept about the same.

The real advantage of automation is to permit the vehicle size to be reduced enough to keep the daily average load factor high. To maintain a high load factor, one must not only reduce the vehicle size but one must place the stations offline so that the vehicles need move only on demand. Since about 95% of urban trips are taken by groups of one, two or three people; since people prefer not to wait very long; and since people generally prefer to ride with their own traveling companions; the use of three-passenger vehicles, off-line stations and demand-responsive service, possible only with automation, provide a very desirable service while maintaining a high daily average load factor, thus reducing energy use. See Anderson (1986).

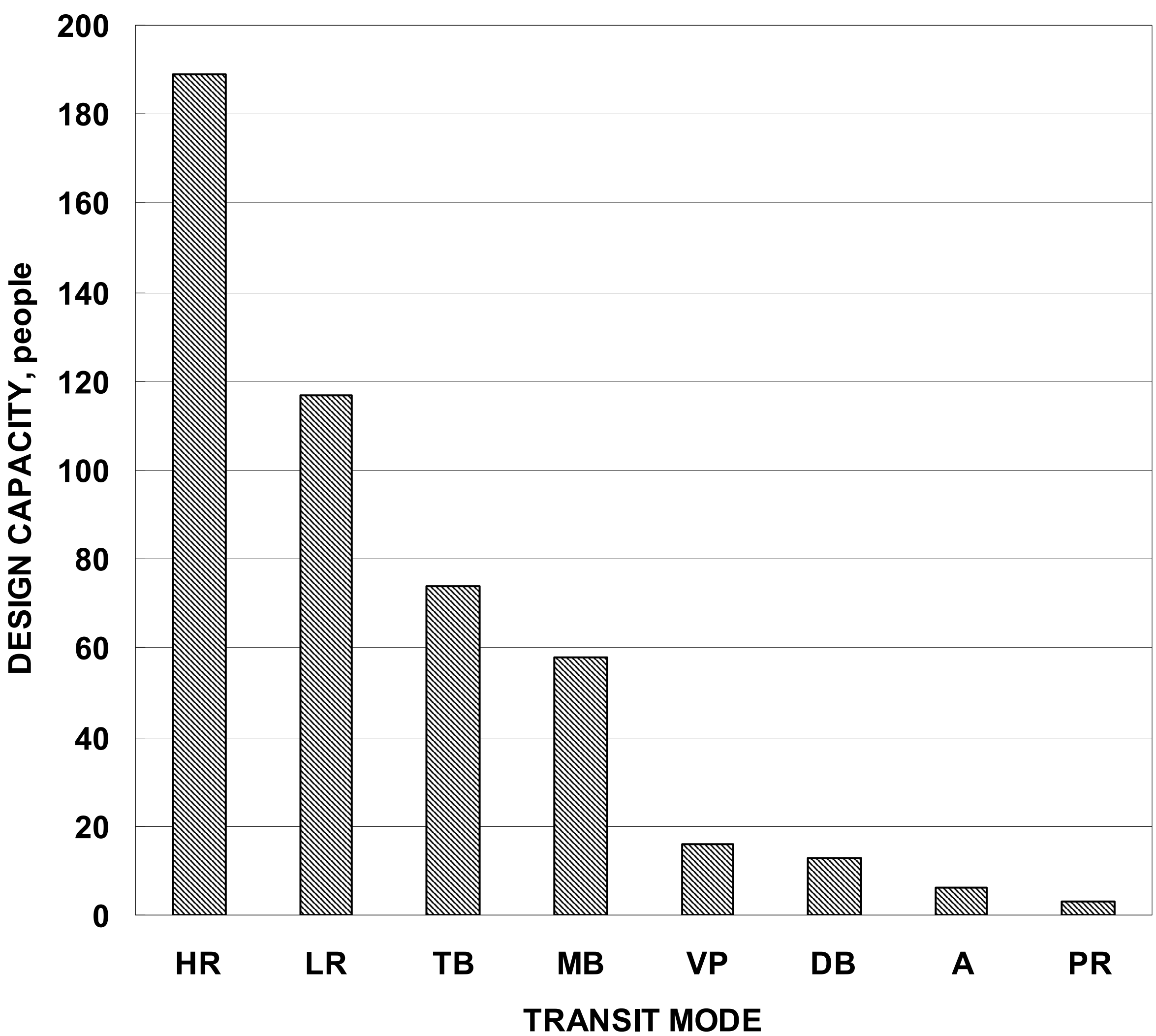

Figure 2. Design Capacity of Transit Vehicles.

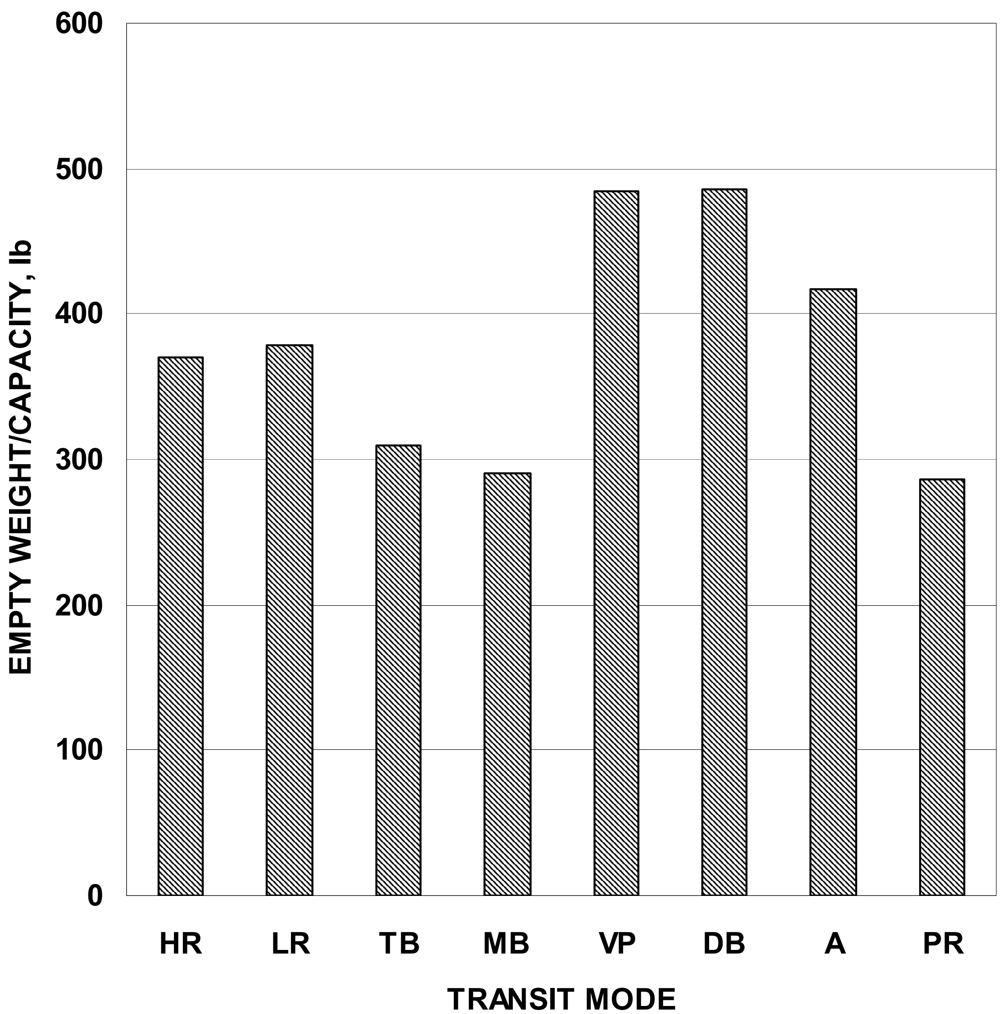

Figure 3. Empty Vehicle Weight per Unit of Capacity.

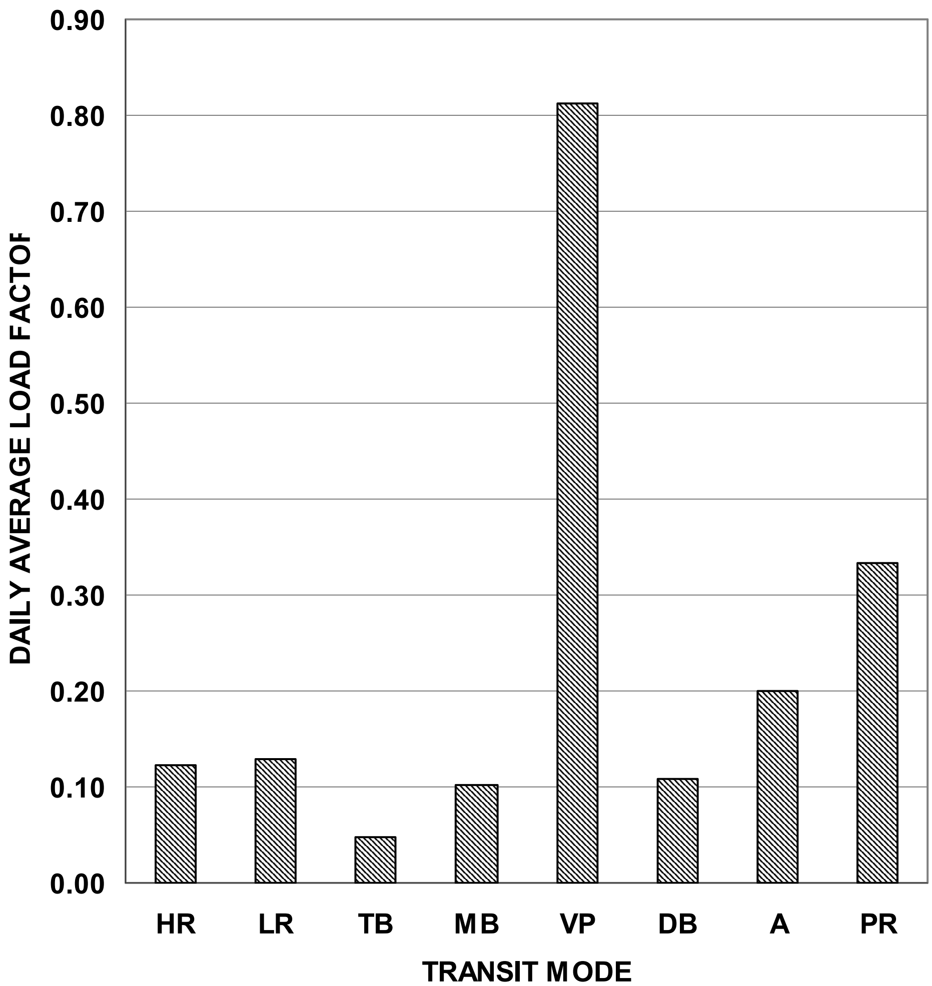

Figure 4. Daily Average Load Factor

The van pool is an exception to the need to provide automation to keep the load factor high. But van pools are designed to be profitable by keeping the seats filled by using them only for the work trip. They cannot be the basis for a general public system. Attempts to do that result in dial-a-bus which averages only 1.4 people per 13-passenger vehicle. But Figure 4 shows that a low load factor can be only one of the reasons that dial-a-bus is the worst energy performer of the eight modes studied (see Figure 11), as other modes have comparably low load factors.

Figure 5 shows the composite of the terms discussed in the above four paragraphs, gross weight per passenger. Here, dial-a-bus stands out, but the trolley bus even more so because of the shockingly low load factors reported. Data on trolley buses in the U. S. is scant and in UMTA (1986) there is data enough to include only two of these systems, those in Boston and Philadelphia.

The kinetic-energy term in the energy equation is the product of gross weight per passenger and the kinetic energy per unit weight per unit distance, shown in Figure 6. This term is a function of distance between stops, comfort acceleration, dwell time, and average speed. Figure 6 shows five curves of kinetic energy plotted as a function of the distance between stops, for five different combinations of comfort acceleration; dwell time and average speed. The one illustrated by hollow squares is for one-eighth g acceleration, 20 second dwell and 10 mph average speed as indicated in the upper right-hand comer. The values for the other curves are listed in the same order. The major lesson of these curves is that, in the region around half-mile distance between stops and below, the specific kinetic-energy term rises very rapidly. If the stations are online, as in the four major conventional modes (HR, LR, TB, MB), reducing station spacing to increase service comes at a substantial energy penalty as well as a substantial reduction in average speed. By use of off-line stations, the conflict is resolved. The distance between stops becomes the trip distance regardless of the station spacing, and the kinetic-energy term remains small.

Air drag depends on the effective frontal area per passenger, the line speed, the wind speed, and to a lesser extent on the dimensionless parameter k given in Appendix A. A wind speed of 15 mph is assumed in all of the calculated results. Figure 7 shows the variation in the frontal area multiplied by the drag coefficient. It is very low for HR because it is determined for a four-vehicle consist. It is very large for DB because of a relatively large frontal area and only 1.4 passengers per vehicle.

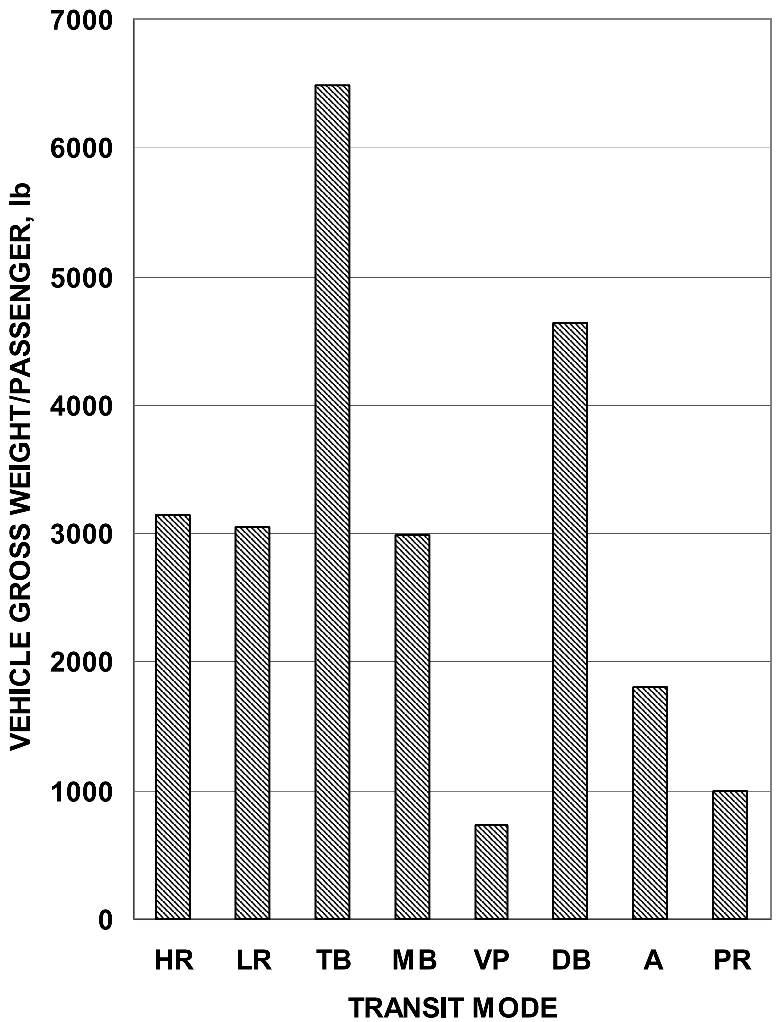

Figure 5. Transit Vehicle Gross Weight per Passenger

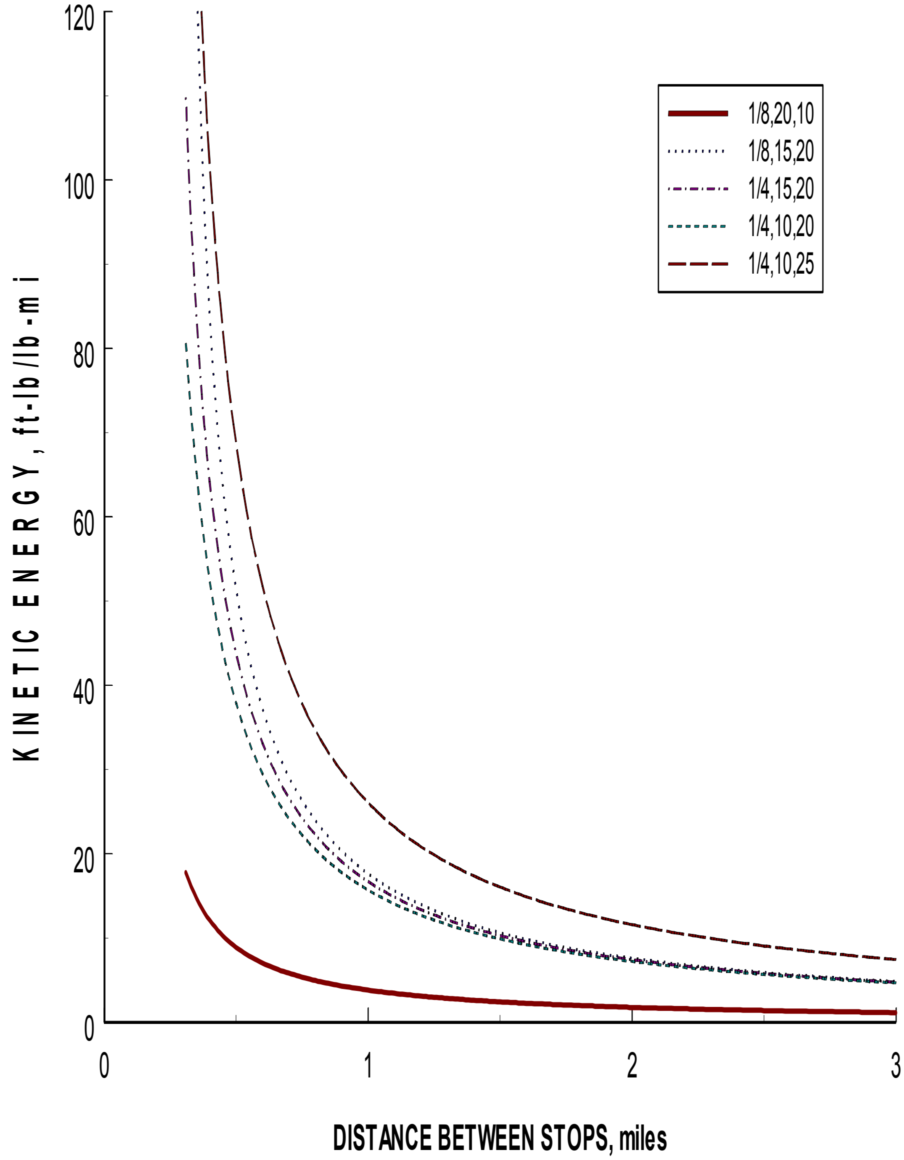

Figure 6. Kinetic Energy per Unit Weight per Unit Distance Between Stops. (Legend is acceleration in g, dwell time in sec, average speed in mph.)

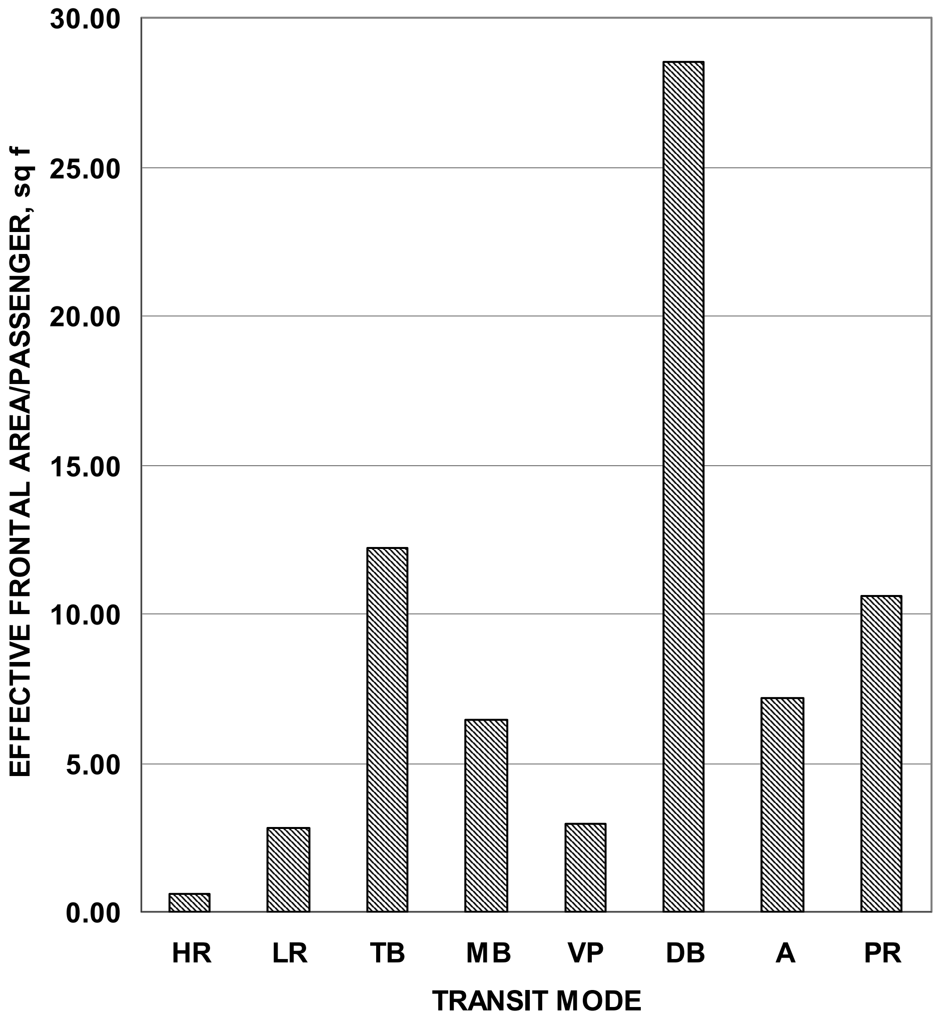

Figure 7. Effective Frontal Area per Passenger.

The average speeds assumed are shown in Figure 8 and, from them, the line speeds in Figure 9. Comparing HR and PR, for example, shows that, while the average speed is 26% higher in PR, the line speed in HR is 76% higher and its square over three times that in PR. Here again the difference is due to a substantial increase in the distance between stops in a given trip made possible by use of off-line stations.

The energy to provide heating and cooling is the power per passenger multipled by trip time per unit trip distance, i.e., divided by average speed. The HVAC power per passenger is shown in Figure 10. The high values correspond to large surface area with few people. VP is best because the load factor is close to one. TB and DB are very large because of their low load factor. PR is larger than either HR or LR, but with its higher average speed this total term is in the same range, as shown in Table 1.

Finally, Figure 11 compares all of the results including construction energy. DB stands out as being a very impractical mode. Indeed, operators of this mode may do well to think of shifting to subsidized taxis, thus substantially reducing the vehicle size. The problem, however, would be that they probably occasionally use these systems at high load factor and in these busy periods going to smaller vehicles means hiring more drivers. Usually about 80% of the operating costs are for driver wages, and, operating on streets, DB is inherently a manually operated system.

LR is the next stand-out. It high kinetic energy is the result of trying to provide reasonable service in both space and time—an impracticality with on-line stopping with or without drivers. High construction energy per passenger-mile compared with HR is a result of much lower ridership on the average of the Toronto systems used in Levinson, et.al.'s (1984) analysis. Quite clearly, the energy use of HR and LR are inherent in the service concept.

Since TB and MB run on city streets, they do not have a large fraction of the construction energy of the running surface accounted to them. Because the damage to roads is proportional to the fourth power of axle loading, this may not be realistic. The large total for TB is mainly due to a load factor half as much as for MB; but, in the auxiliary-energy term, is also due to lower average speed.

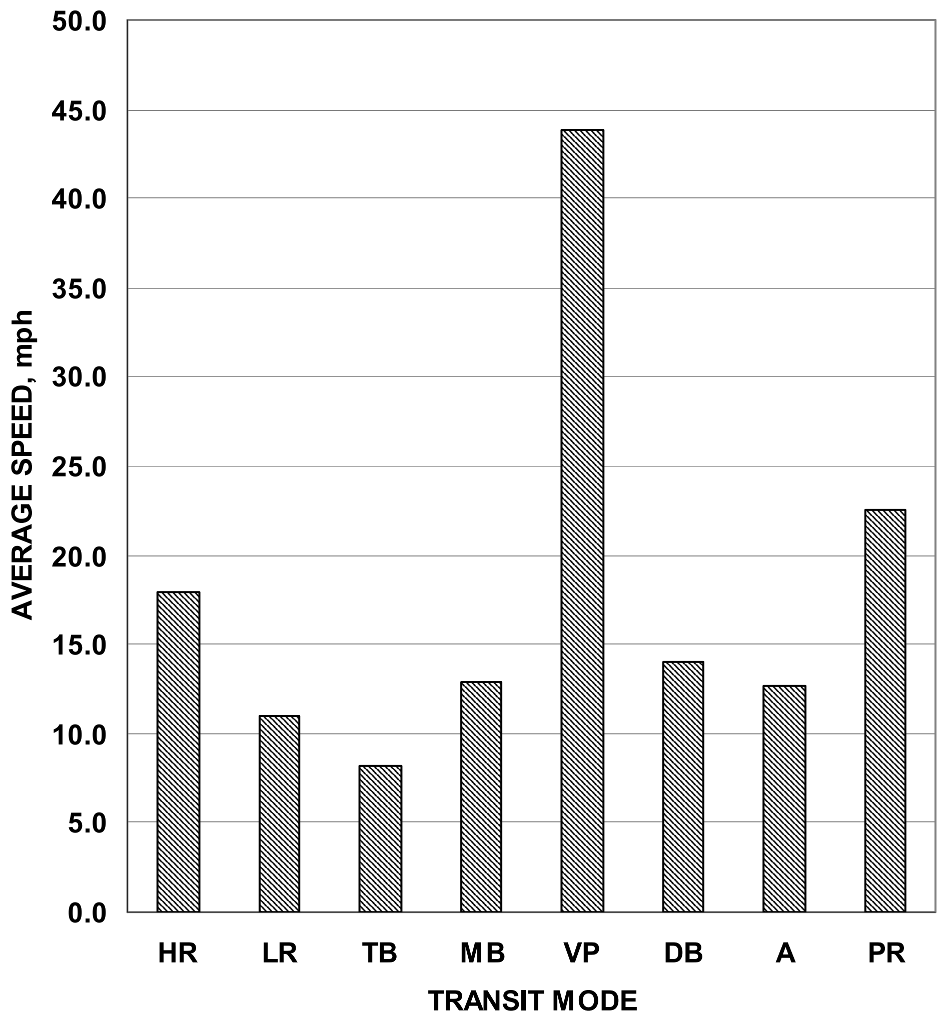

Figure 8. Average Speed.

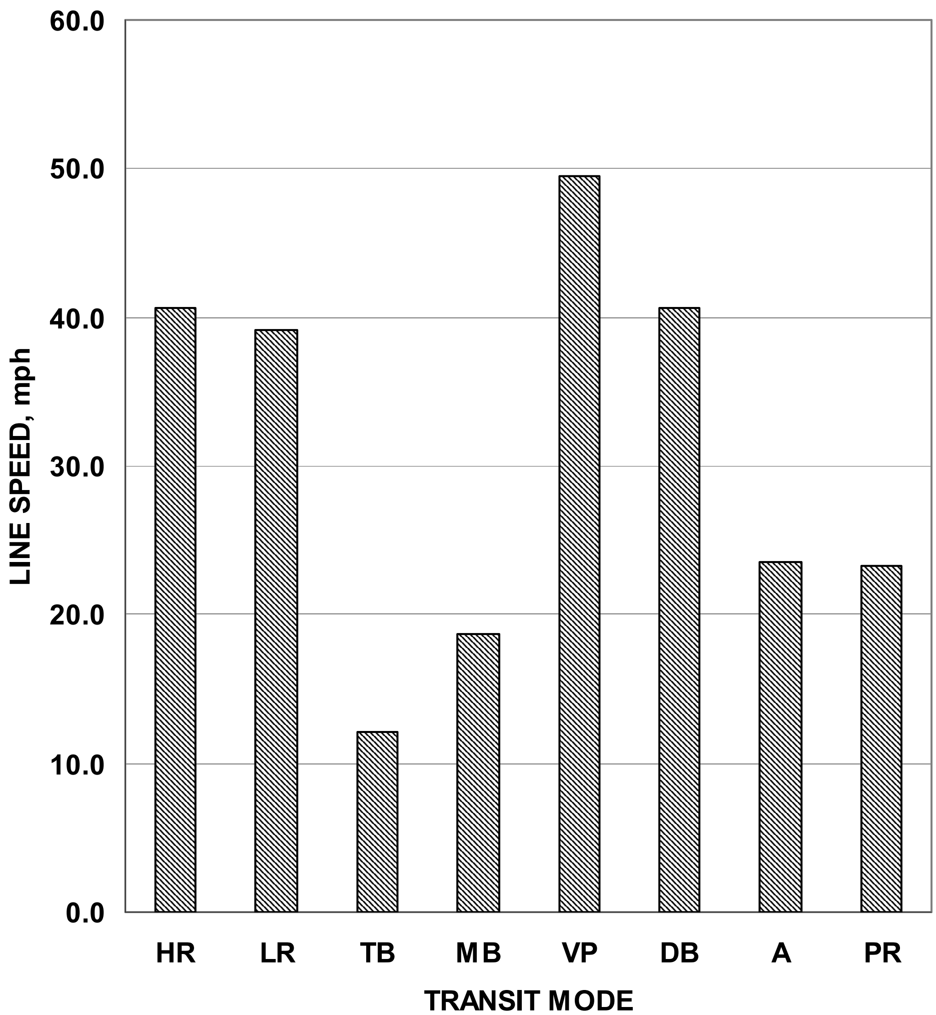

Figure 9. Line Speed.

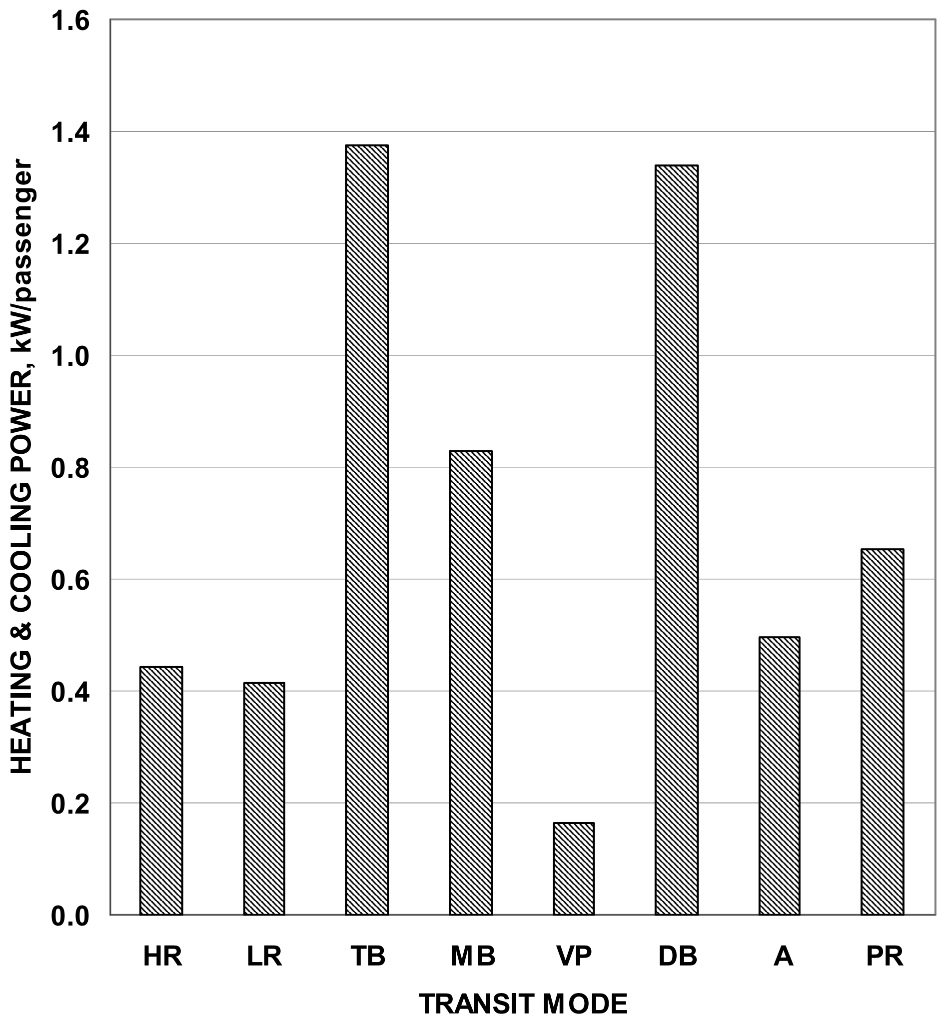

Figure 10. Auxiliary Power per Passenger.

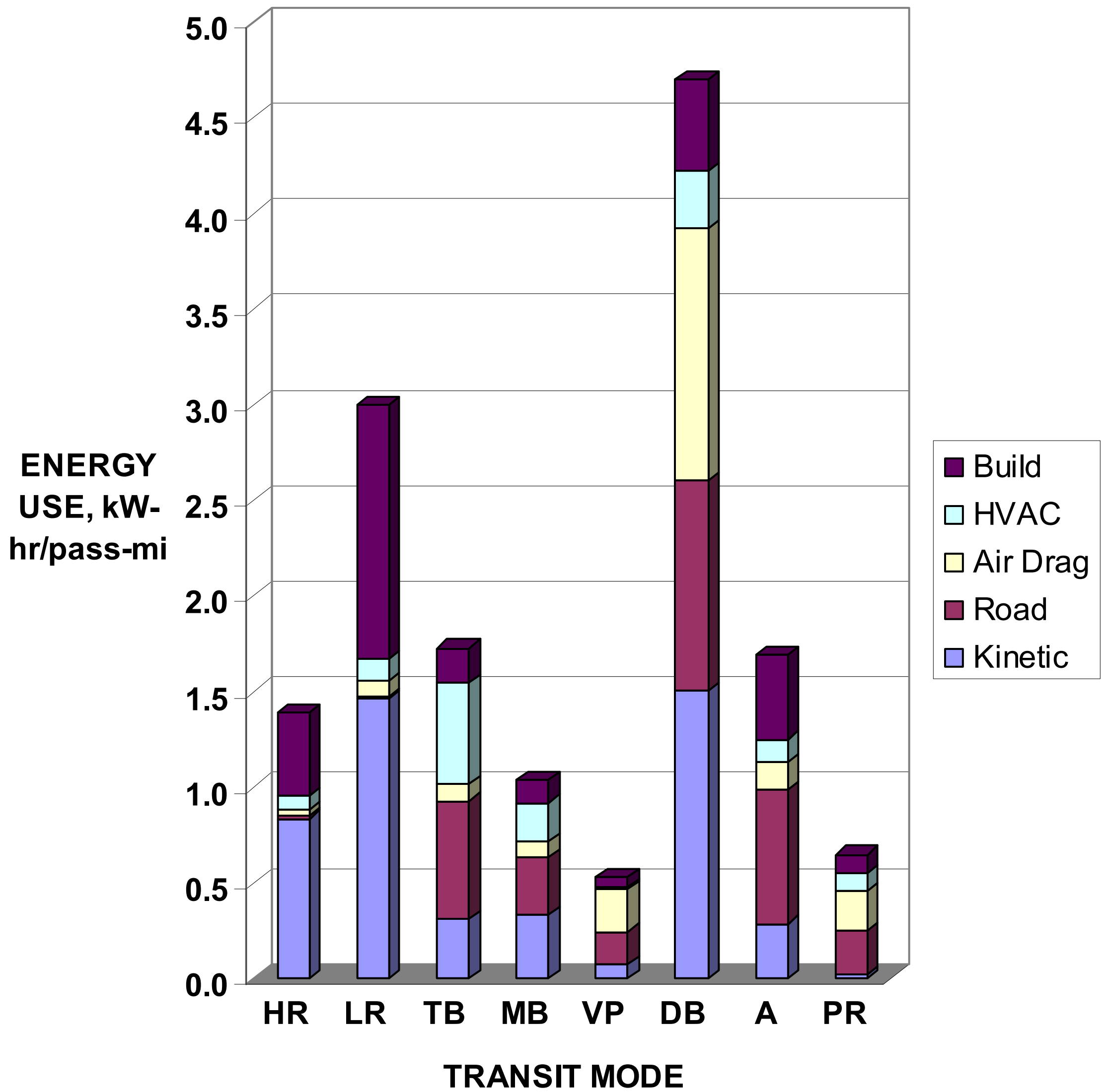

Figure 11. Transit Energy use per Passenger-Mile

VP is the clear winner but is a special-purpose system. It is not practical to think in terms of 12 to 15 passenger vans taking people nonstop directly from origin to destination in any operational mode except from a neighborhood to a specific work location.

In PR, elimination of intermediate stops make the kinetic-energy term almost too small to see. The major effort to further reduce energy use must be in air drag and road resistance. I believe this may be possible but requires a good design and a considerable amount of testing, which is planned. Even though PR has its own guideway structure and stations, the very small size of these structures in an optimized design makes the construction energy quite modest.

Doubling auto gasoline mileage from a fleet average of 20 mph to 40 mpg would make the auto system roughly comparable to personal rail in *direct* energy use, yet so much land is required for the auto that the construction energy is much higher for a given amount of traffic. Because optimized personal rail takes such a small amount of land and can serve as a general urban-transportation system, it provides a practical way to substantially lower transportation energy use.

Conclusions

Based on a general energy equation, the operational energy use of eight modes of urban transportation has been compared. The method employed can easily be programmed on a personal computer and used by the reader with his own set of assumptions. Doing so, one can easily make parameter variations and note the effect on energy use per passenger-mile of variations in the many parameters involved.

Of importance is to note the direction of design changes required to decrease energy use. The clearest direction is to eliminate the intermediate stops by use of off-line stations as it both markedly decreases the kinetic-energy requirement and increases the average speed. Design optimization to minimize the size of the structures and the fleet capacity required, as has been done in the personal rail system mentioned, can markedly reduce the construction-energy component. Since these changes are in the direction of much better service and much less land use, there is no reason for the level of despair about transportation expressed in a recent article by Koepp (1988).

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APPENDIX A

The Transit Energy Equation

Notation

a, b	road resistance coefficients
A_f	frontal area of vehicle
A_m	maximum comfort acceleration
C_D	vehicle drag coefficient
D_s	distance between stops
E_s	energy consumed by a vehicle between stops
F	retarding force on vehicle or train
g	acceleration of gravity
k	$\frac{V_L^2}{2A_m D_s}$, a dimensionless constant
P_{aux}	auxiliary power for heating, air conditioning, lights
p_v	daily average number of passengers per vehicle
t	time variable
t_D	average time vehicle dwells at a station
t_s	time between stops
V	speed variable
V_L	line speed
V_{av}	average speed
V_w	wind speed
W	gross weight of vehicle
ρ	air density
σ_p	propulsion efficiency
σ_u	utility efficiency
σ	overall efficiency, $\sigma_p \sigma_u$

Energy is the integral of power through time, and the power required to move a vehicle is force times speed, or FV. Thus, the energy consumed in moving from rest to cruise speed and back to rest again is given by the equation

$$E_s = \frac{1}{\sigma_p}\int_0^{t_s} FVdt \quad + \quad P_{aux}\frac{D_s}{V_{av}} \tag{A-1}$$

The retarding force F is the sum of the inertia force, the force to overcome air drag, and the force to overcome road resistance, i.e.,

$$F = \frac{W}{g}\frac{dV}{dt} + \frac{1}{2}\rho C_D A_f\left(V^2 + V_w^2\right) + W\left(a + bV\right) \tag{A-2}$$

To solve equation A-1 it is necessary to assume a speed profile.. For a comparative study of transit energy use, it is accurate enough to assume constant acceleration A_m from rest to line (or cruise) speed V_L in time V_L / A_m, followed by cruise at V_L until, beginning at D_s / V_L the vehicle or train decelerates to rest. For this paper I assume that any energy required for braking or returned from regenerative braking is negligible. Noting that during constant acceleration $dt = dV / A_m$, equation A-1 integrates to

$$E_s = \frac{1}{\sigma_p}\left\{\begin{array}{l} \dfrac{WV_L^2}{2g} + \dfrac{1}{A_m}\left[\dfrac{1}{2}\rho C_D A_f\left(\dfrac{V_L^4}{4} + \dfrac{V_w^2 V_L^2}{2}\right) + W\left(\dfrac{aV_L^2}{2} + \dfrac{bV_L^3}{3}\right)\right] + \\ \left[\dfrac{1}{2}\rho C_D A_f\left(V_L^2 + V_w^2\right) + W\left(a + bV_L\right)\right]V_L\left(\dfrac{D_s}{V_L} - \dfrac{V_L}{A_m}\right) \end{array}\right\} + \frac{P_{aux}D_s}{V_{av}} \tag{A-3}$$

Hence, the energy per unit of distance traveled is

$$\frac{E_s}{D_s} = \frac{1}{\sigma_p}\left\{\frac{WV_L^2}{2gD_s} + \frac{1}{2}\rho C_D A_f\left[V_L^2\left(1 - 1.5k\right) + V_w^2\left(1 - k\right)\right] + W\left[a\left(1 - k\right) + bV_L^2\left(1 - \frac{4}{3}k\right)\right]\right\} + \frac{P_{aux}}{V_{av}} \tag{A-4}$$

For transit systems analysis, it is more useful to compare the energy per passenger-mile $\left(E / pm\right)$ of various systems. Then define

$$W_p = \frac{W}{p_v} = \quad \text{gross vehicle weight per passenger.}$$

$$AD_p = \frac{\rho C_D A_f}{2 p_v} = \quad \text{air drag per passenger at unit speed.}$$

$$P_{aux_p} = \frac{P_{aux}}{p_v} = \quad \text{auxiliary power per passenger}$$

Taking into account the efficiency of the electric utility,

$$E/pm = \frac{1}{\sigma}\left\{\frac{W_p V_L^2}{2gD_s} + AD_p\left[V_L^2(1-1.5k) + V_w^2(1-k)\right] + W_p\left[a(1-k) + bV_L\left(1-\frac{4}{3}k\right)\right]\right\} + \frac{P_{aux_p}}{V_{av}\sigma_u} \tag{A-5}$$

APPENDIX B

Line Speed, Average Speed and Travel Distance

Transit systems differ substantially in their ratio of average speed to line speed and average speed is a more meaningful parameter in comparing transit systems. We must therefore express line speed in terms of average speed. For the velocity profile assumed in Appendix A, the average speed is given by

$$V_{av} = \frac{D_s}{\frac{D_s}{V_L} + \frac{V_L}{A_m} + t_D}. \qquad \text{(B-1)}$$

This is a quadratic equation in V_L and can be rearranged into the form

$$V_L^2 - t^* A_m V_L + A_m D_s = 0 \qquad \text{(B-2)}$$

where

$$t^* = \frac{D_s}{V_{av}} - t_D \qquad \text{(B-3)}$$

From equation B-1, note that for small values of V_L, V_{av} increases with V_L to a maximum value, then decreases. Thus, for given V_{av}, equation B-2 has two roots, the smaller of which is the physically meaningful one. Thus, the solution to equation B-2 can be written in the form

$$V_L = \frac{1}{2} A_m t^* \left[1 - \sqrt{1 - \frac{4D_s}{A_m t^{*2}}} \right]. \qquad \text{(B-4)}$$

Importance of Aerodynamic Drag in a PRT System

J. E. Anderson

Summary

This paper examines the effect of the vehicle drag coefficient on the cost of energy use, the thrust and power requirements of the motors, and the load-requirements of the main and lateral wheels. It is found that the major influence of a poor drag coefficient is in the size of the propulsion system and the wheels. It is shown that a major reduction in drag on the cabin can be achieved if all corner radii on the vehicle are at least 1/6th of the side, back or top width, and, by reference to date accumulated by Hoerner in Reference 4, the front should be rounded and should slope as much as practical. To do so the length of the vehicle should be nine feet. Drag on the chassis is sufficiently important that a streamlined shroud should be placed over it to conceal as many of the rough surfaces as is practical. The main, lateral, and switch wheels as well as the power pickup shoes must of course be exposed.

Introduction

The purpose of this paper is to document the importance or lack thereof of designing the PRT vehicle shape to minimize air drag. Consideration of air drag is important for the following reasons:

- To minimize energy costs and hence operating cost.
- To minimize the thrust that must be supplied by the motors.
- To minimize the maximum electrical power that must be supplied by the on-board drive system.
- To minimize the maximum loads on the tires.

The paper documents

1. The energy use, retarding force, and required power.
2. The effect of a practical range of air-drag coefficients.
3. Vehicle features that will reduce drag, and the degree to which the drag coefficients can practically be reduced.

Energy Equations

The retarding force F on an average vehicle in a fleet of vehicles moving in all directions is given by the equation

$$F = \frac{W}{g}\frac{dV}{dt} + \frac{1}{2}\rho C_D A_f (V^2 + V_w^2) + W(a + bV) \tag{1}$$

in which

W	=	vehicle weight
g	=	acceleration of gravity
V	=	vehicle speed
t	=	time

ρ	=	mass of air per unit of volume.
C_D	=	drag coefficient
A_f	=	frontal area
V_w	=	wind speed
a	=	road resistance per unit of weight, independent of speed
b	=	road resistance per unit of weight per unit of speed

For a single vehicle the total wind speed seen by the vehicle is $V + V_w$, but the average of the square of the total wind speed over a fleet of vehicles is

$$\left\langle (V + V_w)^2 \right\rangle = V^2 + 2V\left\langle V_w \right\rangle + \left\langle V_w^2 \right\rangle = V^2 + V_w^2$$

in which the $\left\langle X \right\rangle$ designate the average of the quantity X. With vehicles traveling in all directions $\left\langle V_w \right\rangle = 0$.

The power required to overcome the retarding force is simply

$$Power = FV \tag{2}$$

and the direct energy consumed in moving a vehicle from rest line speed V_L and back to rest again is

$$E_s = (1/\sigma_p)\int_0^{t_s} FVdt + \frac{P_{aux}D_s}{V_{av}} \tag{3}$$

in which

σ_p	=	propulsion efficiency
t_s	=	trip time
D_s	=	trip distance
P_{aux}	=	auxiliary power required for heating, air conditioning, lighting, and equipment
V_{av}	=	average speed, where

$$V_{av} = \frac{D_s}{D_s / V_L + V_L / A_m + t_D} \tag{4}$$

in which

A_m	=	maximum acceleration, and
t_D	=	average station dwell time.

In this calculation an additional jerk term can be safely neglected.

In a system run on electricity, the electrical energy that must be supplied per passenger-mile is found by dividing equation (3) by the trip distance D_s and the average number of people per vehicle p_v. From Appendix A, Reference 1, the electrical energy required per passenger-mile can be written in the form

$$E / pm \quad = \quad E_k + E_a + E_r + E_x \tag{5}$$

in which the kinetic-energy component is

$$E_k = \frac{1}{\sigma_p}\left(\frac{W_p V_L^2}{2gD_s}\right) \tag{6}$$

the air-drag component is

$$E_a = \frac{1}{\sigma_p}\left\{AD_p\left[V_L^2(1-1.5k)+V_w^2(1-k)\right]\right\} \tag{7}$$

the road-resistance component is

$$E_r = \frac{1}{\sigma_p}\left\{W_p\left[a\left(1-k\right)+bV_L\left(1-\frac{4}{3}k\right)\right]\right\} \tag{8}$$

and the auxiliary-energy component is

$$E_x = \frac{P_{aux}}{V_{av}p_v} \tag{9}$$

in which

$W_p \quad = \quad W/p_v$

$AD_p \quad = \quad 0.5\rho C_D A_f / p_v$

The energy required from a primary source (oil, gas, coal, nuclear or solar) is found by dividing equation (5) by the utility efficiency σ_u.

Results

The above equations are solved in the Excel spreadsheet of Reference 2, and all of the parameters for a PRT vehicle are listed there. Figures 1 and 2 graph the solution to equation (5) with the components of equations (6) through (9) broken out separately for two cases: The cabin drag coefficient is taken as 0.33 in both cases. In Figure 1 the chassis drag coefficient is 2 and in Figure 2 the chassis drag coefficient is 1. As expected, air drag becomes the predominate energy consumer more so as line speed increases, and the auxiliary energy, which depends only on trip time, decreases with speed.

The effective drag coefficient of the whole vehicle is

$$C_{D_{veh}} = \frac{C_{D_{cabin}}A_{cabin} + C_{D_{chassis}}A_{chassis}}{A_{cabin} + A_{chassis}} \tag{10}$$

Using the data of Reference 2, the effective drag coefficient becomes

$$C_{D_{veh}} = 0.82C_{D_{cabin}} + 0.18C_{D_{chassis}}$$

or with $C_{D_{cabin}} = 0.33$ and $C_{D_{chassis}} = 2$, $C_{D_{veh}} = 0.271 + 0.360 = 0.63$. So even with only 18% of the total frontal area, chassis dominates the overall drag. If the chassis drag coefficient can be reduced to 1, then $C_{D_{veh}} = 0.271 + 0.180 = 0.45$. Thus it will be worthwhile to place a streamlined shroud over as much of the chassis as practical to reduce its drag.

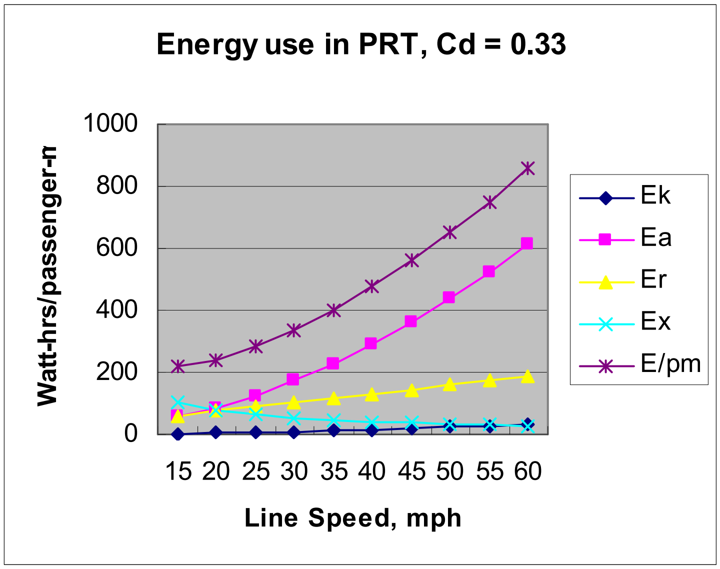

Figure 1. Energy Use with chassis drag coefficient of 2.

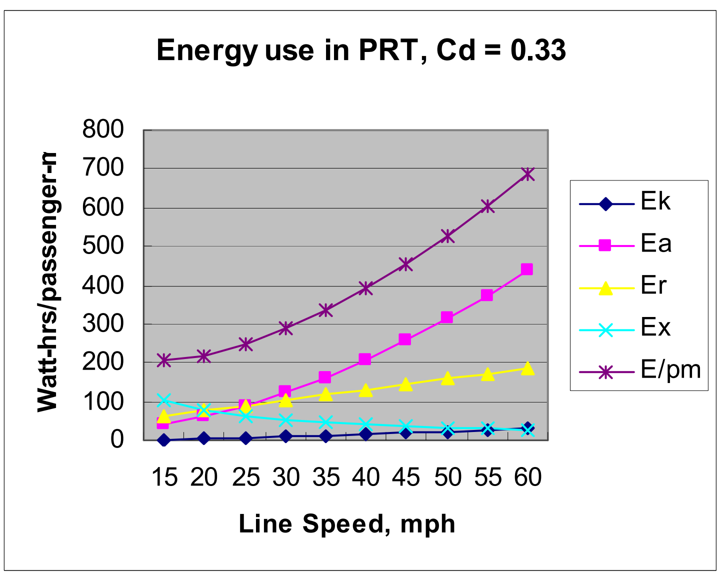

Figure 2. Energy Use with chassis drag coefficient of 1.

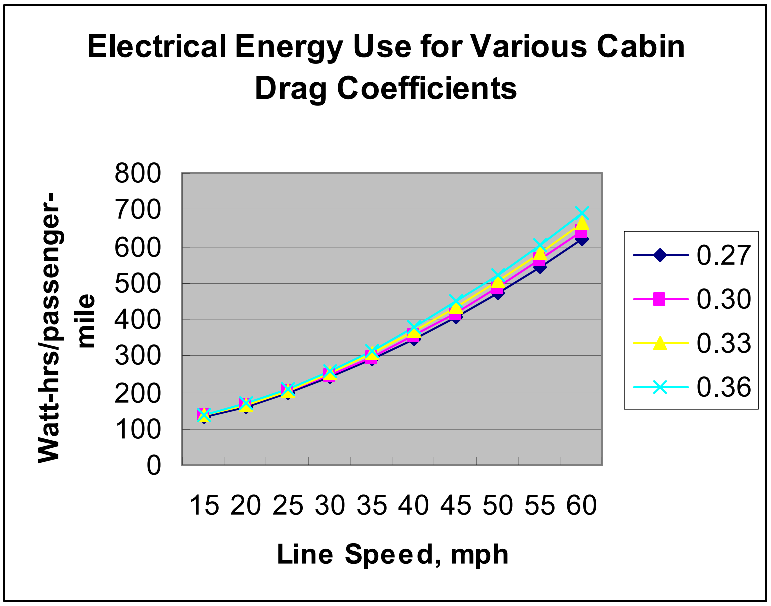

Figure 3.

Figure 3 shows the effect of varying the cabin drag coefficient on the energy use per passenger-mile assuming the chassis drag coefficient is 1. Note that the spread is quite small.

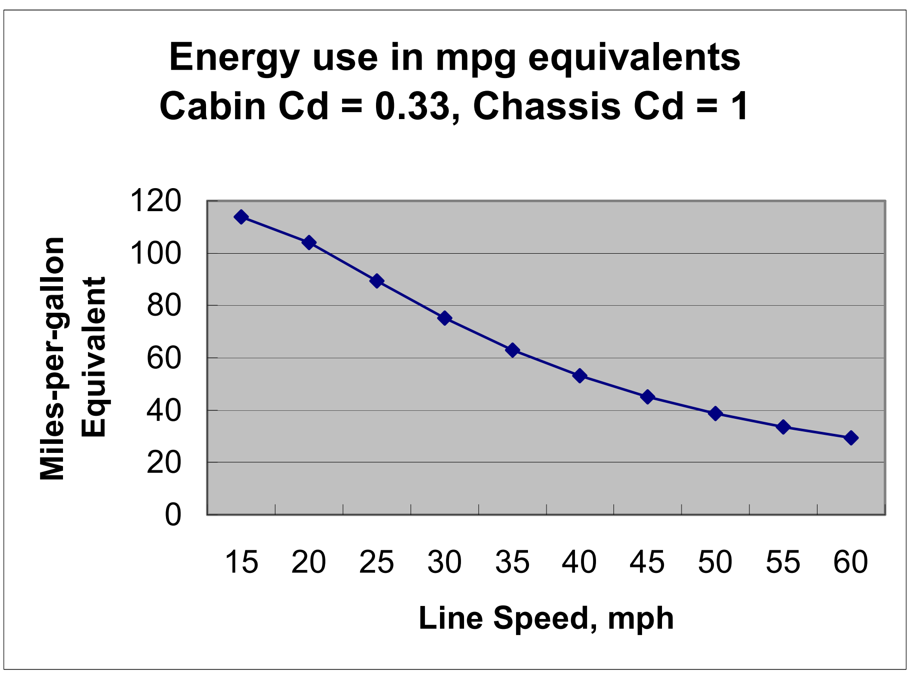

Figure 4.

Figure 4 shows the efficiency of a PRT system in miles-per-gallon equivalent for various line speeds. The meaning is that a barrel of oil in one case is refined into gasoline and used to propel automobiles and the second case is burned in a power plant to make electricity, which is used to drive PRT vehicles. The conversion factors and efficiencies are given in Reference 2. Since the auto system averages about 20 miles per gallon of gasoline, a PRT system with a line speed of 28 mph is equivalent to an automobile system averaging four times as many mpg, but if the line speed in the system is increased to 40 mph, its energy efficiency is equivalent to an auto system averaging about 54 mph, or 2.7 times as many mph as an auto system.

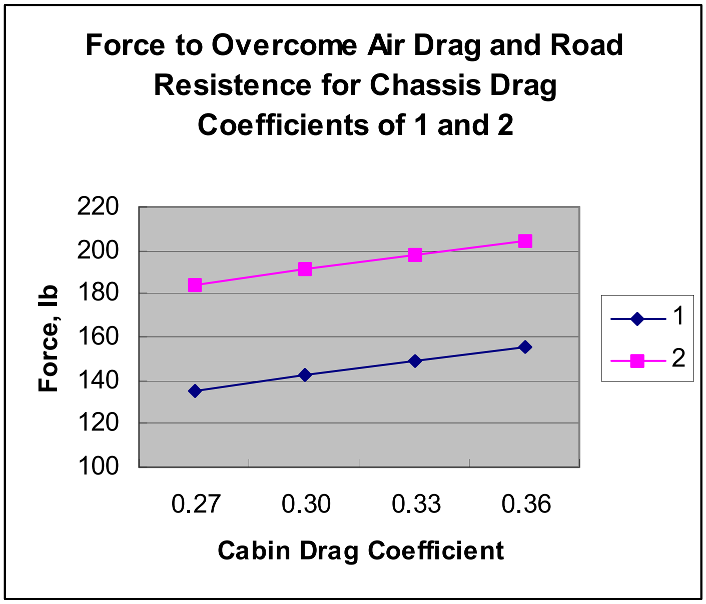

Figure 5

Figure 5 shows the force required to overcome air drag if the vehicle is moving at 40 mph into a 20 mph headwind, since the system will be specified so that the sum of line speed and wind speed will not exceed 60 mph. So it is more important to try to lower the chassis drag coefficient than the cabin drag coefficient.

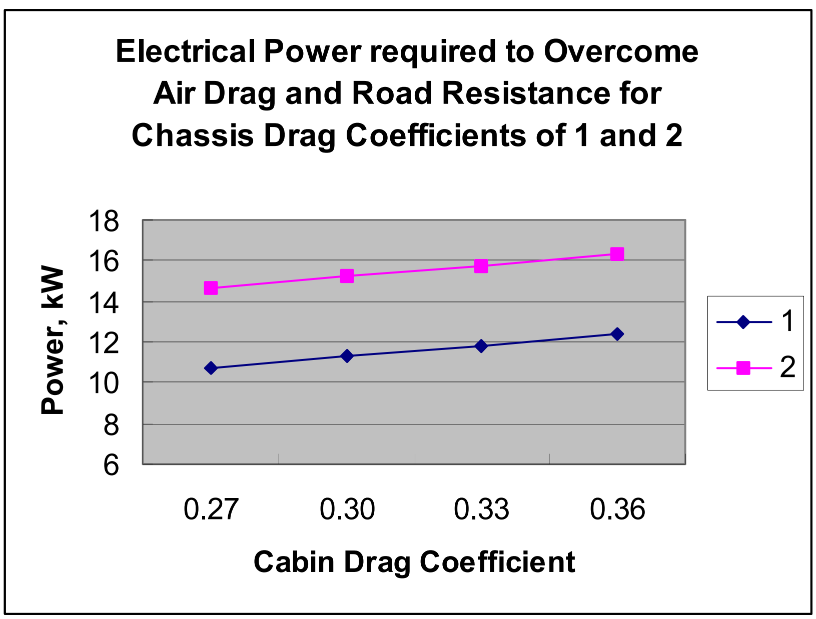

Figure 6.

Figure 6 shows the power required to overcome air drag for the most extreme vehicle and wind speed condition, for various cabin drag coefficients and chassis drag coefficients of 1 and 2, assuming the forces shown in Figure 5. Since the force to overcome air drag is proportional to the square of the sum of line speed and wind speed, and is limited to 60 mph; the power required to overcome air drag is the same for any combination of vehicle speed and wind speed that equals 60 mph. But the power to overcome road resistance increases with vehicle speed independent of wind speed, so if 40 mph is the maximum vehicle speed, that speed with a wind speed of 20 mph gives the greatest power requirement. A conclusion from Figure 6 is that we must if possible strive harder to lower the chassis drag coefficient than the cabin drag coefficient.

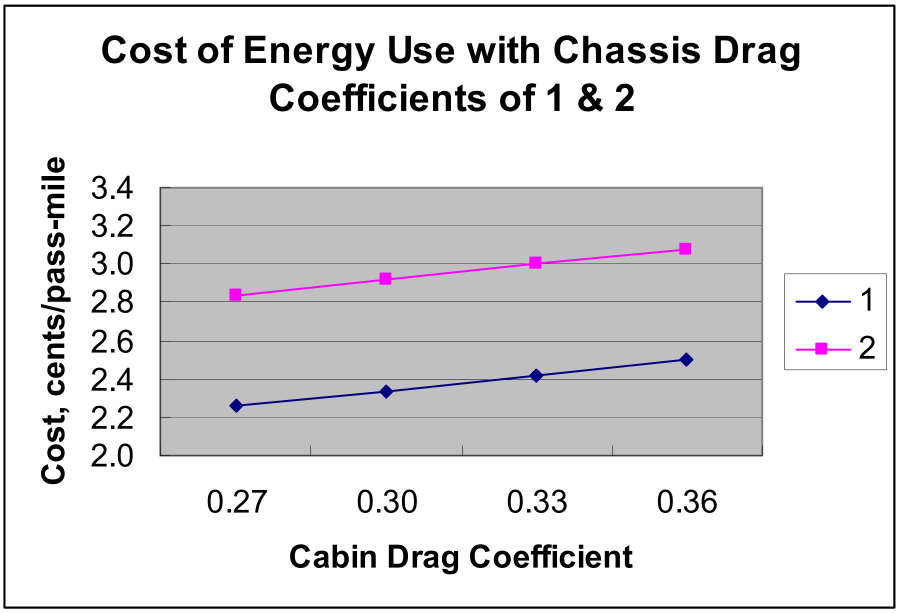

Figure 7.

Figure 7 shows the cost of energy per passenger-mile in a PRT system if the cost of electricity is 6 cents per kW-hr, under the assumption that the line speed is 40 mph and the wind speed is 20 mph. So it is seen that if it would be possible to reduce the chassis drag coefficient from 2 to 1, the cost savings would be about 0.6 cents per passenger-mile, but over the indicated range of cabin drag coefficients, the cost savings is at most about a quarter of a cent per passenger-mile.

Effect of Side-Wind Drag on Wheel Forces

The vehicles must be designed to operate in winds up to 50 mph, which may be crosswinds. Therefore the strength required of the wheel assemblies increases as the side-drag coefficient increases. To determine the loads on the wheel assemblies, I made several runs with my lateral simulation program, LATERAL.BAS, which provides an accurate simulation of the motion of the vehicle as it passes through a merge or diverge section of the guideway subject to the combined side loads of wind, centrifugal force and maximum passenger load. The results of the simulations of a vehicle merging from a curve are shown in the Table 1. Table 2 shows the maximum of the side-wheel loads and the load ratios. Thus with a drag coefficient of 0.6 the maximum side-wheel load is 33% smaller than with a drag coefficient of 1, and twice as much with a drag coefficient of 2 than of 1. Table 3 shows similar results for the main tires, and it is seen that here the influence of the drag coefficient is diminished.

Note that the loads on the main wheels on the windward side diminish as the wind force increases, and indeed when the leeward main-wheel loads reach their maximum, the opposite pair of wheels carry no load at all, but the moment that resists the wind force is resisted by a couple provided by forces on the upper-lateral wheels and the switch wheels. When these forces are the greatest, the lower lateral wheels carry no load at all.

The clear conclusion of this analysis is that reduction in side drag is very important to minimize the loads on the side wheels. On the other hand, the weight and cost of larger lateral wheels is not great – lower side drag just gives a greater margin of safety.

Table 1. Loads on the Wheels of a PRT Vehicle in pounds.

C_d	Switch front	Switch rear	Lower Lateral	Upper front	Lateral rear	Rear left	Main right	Front left	Main right
0.6	420	435	425	530	540	339	856	339	856
1	618	608	596	805	736	311	925	311	925
2	1356	1418	974	1584	1637	455	925	455	925

Table 2.

C_d	Maximum Side-Wheel Load, lb	Load Ratio
0.6	540	0.67
1	805	1
2	1637	2.03

Table 3.

C_d	Maximum Main-Wheel Load, lb	Load Ratio
0.6	856	0.93
1	925	1
2	925	1

Some Results of Investigations to Reduce the Drag Coefficient

References 3 and 4 collect a great deal of data on drag coefficients. The most relevant results required here are shown in Figures 8 and 9. Figure 8 is a reproduction of a figure taken from Reference 3, which was prepared as part of a $1.5 million PRT design study sponsored by the Chicago RTA. It shows the influence of Reynolds Number on the drag on square prisms with the ratio of corner radii to diameter, r/d = 0.021, 0.167, 0.333, and 0.5 (a circle). The Reynolds Number is

$$\mathrm{Re} = \frac{Vd}{\nu}$$

where V is the speed of the air striking the prism, d is its diameter and ν is the kinematic viscosity of air. For a three-foot diameter prism in an air flow of 50 mph, **Re** = $1.4(10)^6$, so for a ratio r/d = $1/6^{th}$, the drag coefficient has sharply dropped from 1 to 0.5. We must of course design our guideway and vehicle for all wind speeds up to a specified maximum, which is 60 mph with vehicles on the guideway and 120 mph with the guideway empty. Note that in the curve for r/d = $1/6^{th}$, the drop in drag coefficient occurs at about Re = $0.7(10)^6$, corresponding to a wind speed of 25 mph. So the ratio of drag at 60 mph to the drag at 25 mph is

$$\frac{C_D V^2\big|_{60mph}}{C_D V^2\big|_{25mph}} = \frac{0.6}{1}\left(\frac{60}{25}\right)^2 = 3.5$$

So, the drag force at the higher speed is greater notwithstanding the lower drag coefficient. The important point is that if the guideway is designed with corner radii at least 1/6th the diameter, the design need use a drag coefficient less than 1, whereas if the corners were sharp, the drag coefficient is 2. Note that larger corner radii don't make much difference.

Figure 9 helps illustrate why the corner radii help so much up to a point and not much beyond. These diagrams, developed with computational fluid dynamics, show that with sharp corners, the region of flow disturbed is quite large, and boundary-layer separation occurs at the front edges, producing large swirling eddies, which are seen as barriers to the flow ahead of the prism. With corner radii of 1/6th the diameter, boundary-layer separation is delayed until to rear edges, thus creating a flow field much lest disturbed above and below the prism, and would lead one to guess that perhaps the drag coefficient could be as low as 1. But why does it go below 1? To fully understand why requires deeper understanding of aerodynamics than can be entered into here, but the effect is as if the air stream "sees" an object smaller than its physical dimensions, and explains why very streamlined bodies can have drag coefficients less than one tenth. The important factor is that nature likes substantially rounded corners. Reference 4, and many other works of this type, show a great deal of experimental data on shapes that minimize drag. It is important that not only the surface facing the airflow must be streamlined, but that the way the leeward side tapers off or rounds is important too. One book on automotive design (unfortunately I don't have the reference available) concludes that the rear should be cut off at an area about half the maximum area, which implies that the rear height and width dimensions should be about $0.5^{1/2}$ = 0/707 times the maximum. Half that top and bottom implies a corner radius to height ratio of (1-0.707)/2 = 0.146, which is a little less than 1/6th! Figure 10 is taken from Chapter XII, page 12-3 of Reference 4. It shows measured drag coefficients of a six different vehicle shapes, and can serve as a guide for drag reduction of the PRT vehicle cabin. Data on how to reduce drag on the chassis, which is of course constrained inside a tube is not so easy to find, but the general principles suggest that it is necessary to place around it a streamlined shroud.

August 16, 1991

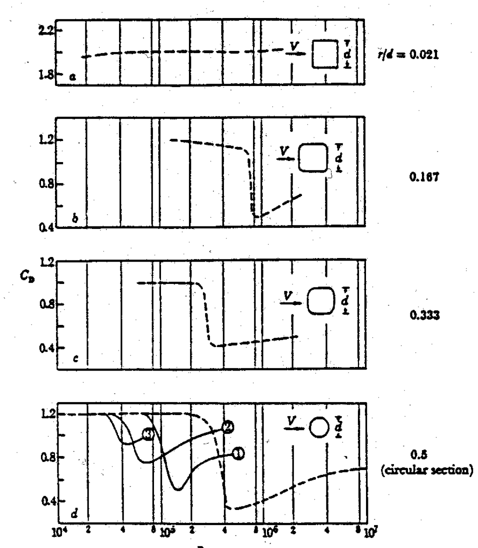

FIGURE 3. Influence of Reynolds number, corner radius, and surface roughness on the values of C_D for prisms of square section and circular cylinders (a.r. = ∞). ---, smooth surface; ——, sanded surface. In 3d: (1) $k/d = 0.002$; (2) $k/d = 0.007$; (3) $k/d = 0.020$ (where k is the grain size of sand).

FIGURE N-17 (REF. 41)

Figure 8.

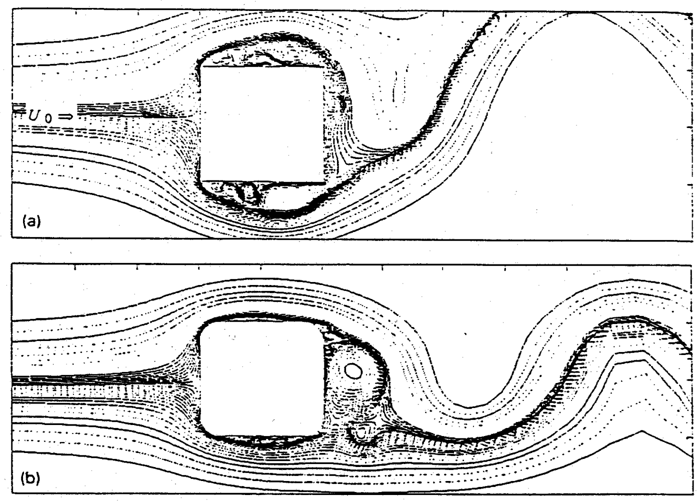

Fig. 4. Computed instantaneous streamlines around a square cylinder at $Re = 10^5$: (a) sharp corners ($r/B = 0.0$); (b) rounded corners ($r/B = 1/6$).

FIGURE N-24 (REF. 54)

Figure 9.

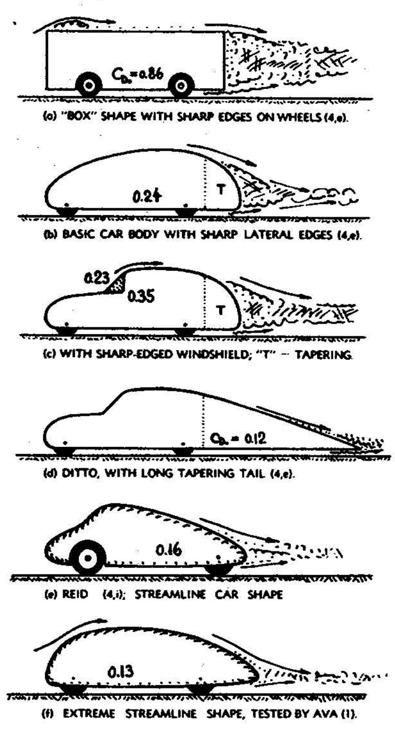

Figure 4. Drag coefficients of several smooth wind tunnel models (tested over fixed ground plate).

Figure 10.

The Cost of Drag Reduction

Study of well-known data on auto drag coefficients shows that a major factor in reducing the drag coefficient of the vehicle is to give it more tapered front and rear ends, but do so requires that the vehicle be longer, and hence somewhat heavier, and that the station platforms be correspondingly longer. But the defining load in design of the guideway is fully loaded vehicles nose to tail. If the extra length required to produce the desired aerodynamic shape has a weight per unit length less than the rest of the vehicle, then the smeared out uniform load that defines the design is reduced, i.e., longer vehicles with lightweight aerodynamic nose cones reduce the required weight of the guideway by a small amount.

The main cost penalty is in the extra length required of the station platform and the off-line guideway. A typical system might have say five-berth stations every half-mile, or ten station berths per mile. But for every berth, there must be a waiting position, so the off-line guideway must have 20 locations for vehicles to stop per mile. If the extra length is say half a foot, then there must be 10 more feet of guideway per mile or 100(10/5280) = 0.2% more guideway, which translates to an increased budget for guideways of 0.2%. If the guideway costs $2,600,000 per mile, as estimated in CostEstimate.xls, the extra cost is $4900 per mile. Consider the extra station cost. If, as calculated in CostEstimate.xls, a 5-berth station, not counting the equipment, costs $100,000, then if each berth is 9 ft long, an extra half a foot increases the cost by (0.5/9)(100,000) = $5,500. With 2 stations per mile, that would be an increased cost of $11,000 per mile. Thus the total cost increase due to increasing the vehicle length by half a foot is about $16,000 per mile. Out of a total system cost of say $7,000,000 per mile, this is an increase of 0.23%, which is in the noise of the cost estimate.

The major cost increase due to a poor vehicle drag coefficient is likely to be in the increased cost of the LIMs and variable-frequency drives needed to run them and in the required heavier lateral wheels. These are not insignificant factors, but the cost increase due to higher drag is not easy to estimate at this time. It is manifested in both increased cost of on-board components, but increased vehicle weight, which in itself increases energy use and would somewhat increase the required weight of the guideway. A major problem is that the drag coefficient can be estimated only roughly in advance of operation of the test track, where by means of coasting tests, the drag coefficient as well as the road-resistance coefficients can be determined. Thus it is prudent to use knowledge available to minimize vehicle drag as much as is practical, given constraints such as the design and operation of the door.

Conclusion

The PRT vehicle should be designed to be 9 ft long with a smoothly varying external shape, and the sides and rear corners rounded with a radius of at least 10 inches.

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This paper was developed for an invited lecture while the author was employed full time by Boston University. He also donated his time to Taxi 2000 Corporation.